Math 216 Exam #2 Solutions

Fall 2004 - Hartlaub.

#1 (a) $$X_{ij} = \theta + \beta_i + \tau_j + e_{ij} \qquad \begin{array}{l} i=1,\dots,9 \\ j=1,2,3 \end{array}$$

X_{ij} = resting metabolic rate for the i^{th} subject in protocol j

θ = overall median

β_i = block (subject) i effect

τ_j = treatment (protocol) j effect

e_{ij} = random errors from a continuous distribution with median 0.

(b)

$H_0: [\tau_1 = \tau_2 = \tau_3]$

vs H_1: [τ_1, τ_2, and τ_3 are not all equal].

(c) Use Friedman's Test : Stat > Nonparametrics > Friedman

$S = 0.22$ d.f. $= 2$ $P = 0.895$

Protocol 1 ;	$R_1 = 17$	7687.3
Protocol 2 :	$R_2 = 19$	7884.7
Potocol 3 ;	$R_3 = 18$	7831.0
	sum of ranks	estimated medians

Conclusion: Since .895 > .05, we cannot reject H_0.
i.e., RMR measurements are <u>not</u> significantly different for the three protocols.

(d) No. There is no need for follow-up multiple comparisons because the RMRs are not different. The purpose of multiple comparisons procedures is to identify significant differences.

(e) YES. Since the measurements do NOt depend on the protocol, comparisons of the results of different studies conducted by different laboratories using different protocols can be made

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#1 (f) $H_0 : [\tau = 0]$ vs $H_1 : [\tau > 0]$,

where τ = Kendall's population correlation coefficient

$= 2P\{(Y_2 - Y_1)(X_2 - X_1) > 0\} - 1.$

sorted

Protocol 1	Protocol 3	Concordant	Discordant
6921	7132	~~IIII~~ I	II
7046	6939	~~IIII~~ II	
7131	7095	~~IIII~~ I	
7249	7471	~~IIII~~	
7715	7831	IIII	
7812	8179	III	
8062	8685	II	
9551	8840	I	
9862	9711		

$$\boxed{K = 34 - 2 = 32}$$

Table A.30 $n = 9$

$\alpha = .012 \Rightarrow K_{.012} = 22$

Since $K = 32 > K_{.012} = 22$, we reject H_0 at the .012 level.

According to Table A.30, the p-value $= P(K \geq 32) = 0$.

Since this p-value is smaller than any significance level, we reject H_0 and conclude that there is a statistically significant positive association between the RMRs for protocols 1 and 3.

Point Estimate: $\hat{\tau} = \dfrac{2K}{n(n-1)} = \dfrac{2(32)}{9(8)} = .8889$

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#2. (a) $X_{ij} = \theta + \tau_j + e_{ij}$ $i = 1, \ldots, 10$; $j = 1, 2, 3$

X_{ij} = bone density for the i^{th} rat in the j^{th} treatment group.

θ = overall median

τ_j = treatment (group) j effect.

e_{ij}'s = random errors from a continuous distribution with median 0.

(b) $H_0: [\tau_1 = \tau_2 = \tau_3]$ vs. $H_1: [\tau_1, \tau_2,$ and τ_3 not all equal$]$

(c) Kruskal-Wallis Test; Stat > Nonparametrics > Kruskal-Wallis

τ_1 = control group
τ_2 = lowjump
τ_3 = high jump

Group	N	Median	Ave Rank	Z
Control	10	601.5	10.2	-2.33
Highjump	10	637	22.7	3.15
Lowjump	10	606	13.7	-0.81

$H = 10.66$ D.f. = 2. $P = .005$

$\boxed{H = 10.68 \quad d.f. = 2. \quad P = .005}$ (adjusted for ties)

Conclusion: Since $.005 < .05$, we reject H_0 and conclude that there is a significant difference in the bone densities for the three groups.

(d) YES, use the Steel-Dwass-Critchlow-Fligner two sided all treatments MC procedure described in Section 6.5.

(e) $H_1: [\tau_C \le \tau_{low} \le \tau_{High}$, with at least one strict inequality$]$.

(f) Jonckheere-Terpstra Test (described in Section 6.2)

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#2. (g.) I would suggest that she use a variety of different intensities of exercise (to form the different groups) and the Mack-Wolfe peak unknown procedure for umbrella alternatives.

$$H_1: [\, \tau_1 \le \cdots \le \tau_{p-1} \le \tau_p \ge \tau_{p+1} \ge \cdots \ge \tau_k,$$
with at least one strict inequality, for some $p \in \{1, \ldots, k\}$]

(h) $\theta = \tau_{control} - \frac{1}{2}(\tau_{low} + \tau_{high})$

Use the Spjøtvoll estimator, decribed in section 6.8:

$$\hat{\theta} = \bar{\Delta}_{control} - \frac{1}{2}\bar{\Delta}_{low} - \frac{1}{2}\bar{\Delta}_{high}$$